CDMA Channel Estimation with Adaptive Fuzzy Filters

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Abstract: This paper proposes a hierarchical system of three adaptive fuzzy filters into the RAKE receiver to estimate the channel coefficients in a downlink multipath fading CDMA channel. The simplest filter is proved to be equivalent with the alpha tracker. In tested circumstances (Rayleigh fading with AWGN) the two simpler filters proved to be most useful. The nested nature of the filters makes it possible to construct a ‘hybrid’ filter according to the situation.

Key-Words: Communication, CDMA, Fuzzy Tracking, Rayleigh Fading, Multipath, RAKE

1 Introduction
In the code-division multiple access (CDMA), the channel estimation is an important task because it considerably increases the accuracy of a system. The fuzzy reasoning offers a one possibility to this problem.

The fuzzy logic dates back on the sixties when Zadeh published his article ‘Fuzzy sets’ [6]. The idea in the fuzzy reasoning is to rule the uncertainty, to make decisions based on insufficient, faulty or contradictory information. The heuristic nature of the fuzzy inference offers a wide range of applications especially in systems, where the system model is complex or sometimes unknown.

In [4] we introduced a fuzzy filter to track the channel coefficients in a fading multipath downlink CDMA channel. In this paper we continue those studies by constructing a hierarchical group of three adaptive filters and by comparing their accuracy to each other. The trackers are implemented in the conventional RAKE receiver and the bit error rate (BER) values are counted in the multipath many users conditions. As a theoretical result it turns out that the simplest type of considered fuzzy filters includes in the channel estimation well known alpha tracker.

2 Data Model
The synchronous data of a user \( k, k = 1,2,\ldots, K \) have the form

\[ x_k(t) = b_k(t)s_k(t) \]  \hspace{1cm} (1)

and the combination of all users equals

\[ x(t) = \sum_{k=1}^{K} b_k(t)s_k(t) \]  \hspace{1cm} (2)

where \( b_k = b_k(t) \in \{ \pm 1 \} \) is the symbol and \( s_k = s_k(t) \in \{ \pm 1 \} \) the signature waveform (code) transmitted by the \( k \)th user, respectively. The processing gain \( C = T / T_c \) (\( T \) is the duration of a symbol and \( T_c \) the chip interval).

In the data channel the signal \( x \) is corrupted by the fading, multiple access interference (MAI) and noise. In addition, due to reflections from buildings, hills, mountains and so on the electromagnetic radiation proceeds along many paths to a receiver’s antenna. The lengths of those paths are different and therefore the faded and noisy signals of each path are delayed and influenced by the inter-path interference (IPI). So the received signal (chip flow) has the form
\[ y(t) = \sum_{i=0}^{L-1} c_i(t) x(t-d_i) + n(t) \quad (3) \]

where \( L \) is the number of paths, \( c_i \) \((i = 0,1,...,L-1)\) time-dependent complex channel coefficients of each path, \( d_i \) the delay of the path \( i \) and \( n(t) \) an additive zero-mean white Gaussian noise (AWGN). By combining (2) and (3) we have

\[ y(t) = \sum_{i=0}^{L-1} c_i(t) b_i(t-d_i) s_i(t-d_i) + n(t) \quad (4) \]

### 3 Simulation Model

The symbols are transmitted in the packets of \( n \) information symbols. In addition, in front and behind of every packet \( m \) auxiliary symbols (unknown to the receiver) are used.

<table>
<thead>
<tr>
<th>( m ) auxiliary symbols</th>
<th>( n ) data symbols</th>
<th>( m ) auxiliary symbols</th>
</tr>
</thead>
</table>

Fig. 1 The structure of a simulation symbol packet.

The sampled chip flow is received. In the reception the conventional RAKE receiver is used and the delays \( d_i \) of each path are assumed to be known. The channel coefficients \( c_i(t) \) are assumed to be constant during the symbol period \( T \).

### 4 Tracking Methods

The tracking methods considered in this paper are iterative in the sense that the estimated symbol \( \hat{b}(i-1) \) is immediately used in the prediction of the coefficient \( c_i(i) \). Especially in cases where the channel is faded, an erroneous estimate \( \hat{b}(i-1) \) (+1 or –1) starts the process in which the signs of the following estimates \( \hat{c}_i(i) \) and symbols \( b(i) \) are systematically changed in a long period. By using differential modulation we can ignore the described phenomena and avoid a series of bit errors. Therefore every symbol is differentially encoded before the spreading.

**Alpha tracker**

In the alpha tracker a channel coefficient \( c_i(i) \) for the \( i \)th symbol is predicted with aid of the previous coefficient estimation \( \hat{c}_i(i-1) \), the previous symbol estimation \( \hat{b}(i-1) \) and measured data \( y_i(i-1) \) separately for each path \( l \):

\[ \hat{c}_i(i) = \hat{c}_i(i-1) + (1-\alpha)\hat{c}_i(i-1) - \hat{c}_i(i-1) \quad (5) \]

where \( \hat{c}_i(i-1) = \hat{b}(i-1)y_i(i-1) \) is the measured coefficient.

The alpha tracker is adjusted by a single parameter only. In noisy circumstances the current estimation in heavily based on the previous estimation (alpha large, the correction term small) and vice versa. The difference \( (\hat{c}_i(i-1) - \hat{c}_i(i-1)) \) is called the error term and it is denoted by \( err(i-1) \).

\[ err(i-1) \rightarrow \alpha \rightarrow d\hat{c}_i(i-1) = (1-\alpha)err(i-1) \quad (6) \]

**Fuzzy tracker**

The fuzzy method applied here is based on two phases. In the first phase for each path the coefficients \( \hat{c}_i(i), i=1,2,...,n \) of a packet are
estimated and the received chips are buffered. As in the case of the alpha tracker, to compute the coefficients \( \hat{c}_i(i) \) the symbols \( \hat{b}(i) \) must recursively be decided symbol-by-symbol. These temporary coefficients and symbols we call here the pre-coefficients and the pre-symbols, respectively.

Because of the tuning of the tracker pre-coefficients \( \hat{c}_i(i) \) are time delayed versions of the true ones. In the second phase this disadvantage is corrected by the suitable opposite time-shift \( s \):

\[
\hat{c}_i(i) = \hat{c}_i(i + s) \quad (6)
\]

The coefficients \( \hat{c}_i(i), i = 1,2,...,n \) together with the buffered chips of the packet under consideration are now used to estimate the final (differentially modulated) symbols:

\[
\hat{b}(i) = \text{sgn}\left( \sum_{i=0}^{n-1} \hat{c}_i(i) s_i(i) y(C(i-1) + i + d_i) \right)
\]

\[
= \text{sgn}\left( \sum_{i=0}^{n-1} \hat{c}_i(i) \sum_{i=1}^{C} s_i(i) y(C(i-1) + i + d_i) \right) \quad (7)
\]

By denoting \( s_k = (s_1(1),...,s_k(C))^T \), \( y_q = (y(C(i-1) + 1 + d_i),...,y(C(i + d_i)))^T \) we have

\[
\hat{b}(i) = \text{sgn}\left( \sum_{i=0}^{n-1} \hat{c}_i(i) s_k^T y_q \right) \quad (8)
\]

The final result can be obtained by demodulating symbols in (8).

5 Hierarchical System of Fuzzy Trackers

In the receiver the first symbol of a packet is simply guessed. The remaining \( m-1 \) auxiliary symbols are determined by using the alpha tracker with fixed alpha.

5.1 Three-input model

First we consider the three-input fuzzy tracker. The first one is the difference of the measured and the predicted coefficients \( \text{err}_i(i-1) = \hat{c}_i(i-1) - \hat{c}_i(i-1) \) (error). The second one is the chance of that difference \( \text{derr}_i(i-1) = \text{err}_i(i-1) - \text{err}_i(i-2) \) (change in error) and the third one is the delayed output of the filter \( \hat{c}_i(i-1) \) (feedback). As an output the tracker gives the correction term \( d\hat{c}_i(i-1) \) for the next coefficient (Figure 4). So we have

\[
\hat{c}_i(i) = \hat{c}_i(i-1) + d\hat{c}_i(i-1) \quad (9)
\]

![Fig. 4 The three-input fuzzy tracker.](image)

**Membership functions**

For the input terms we use the triangle-shape membership functions. For the error term we have

\[
\mu_{e_2} \quad \mu_{e_1} \quad \mu_{e_0} \quad \mu_{c} \quad \mu_{d}\]

![Fig. 5 The triangle-shape membership functions of error term \( n = 2 \)](image)

The membership functions for the change in error \( \mu_{e_2}^c \), \( j = n,...,n \) and for the feedback \( \mu_{e_0}^d \), \( k = n,...,n \) are defined by the analogous way.

For the output we use singletons:

\[
\mu_{c}^z(z) = \begin{cases} 1, & z = k \times \mu_{c}^z \\ 0, & \text{otherwise} \end{cases} \quad (10)
\]
By setting \( dc(p-I) = 0 \) in (12) we have \( \mu_i^d(dc(p-I)) = 1 \), \( \mu_i^e(dc(p-I)) = 0 \), \( k \neq 0 \) which implies

\[
dc(p) = k_j \left( \sum_{j=\infty}^{n} \left( (i + j) \mu_i^e(e(p)) \mu_i^d(dc(p)) \right) \right)
\]

(13)

So the two-input filter is the special case of the three-input one. Furthermore, if \( dc \equiv 0 \) in (13), the two-input model is reduced to a single-input-single-output (SISO) system.

\[
dc(p) = k_j \sum_{i=\infty}^{n} \mu_i^e(e(p))
\]

(14)

Without loss of generality it can be assumed that the input error term is bounded, \( |e(p)| \leq S \) for all \( p \). Let \( k_j \geq S \) in (10). Now we get from (14)

\[
dc(p) = \frac{k_f}{k_i} e(p)
\]

(15)

By choosing \( k_f = 1 - \alpha \) formula (15) gives the output of the alpha tracker. We have proved the following result: for a given alpha tracker it is always possible to construct a single-input fuzzy tracker such that their outputs coincide, i.e. the alpha tracker is a special case of the fuzzy tracker (12).

6 Comparison of Trackers
The three-input fuzzy tracker in Figure 4 includes four adjustable parameters \( k_i, k_2, k_f \) and the time-shift \( s \) (16). In addition, the number of fuzzy rules can vary. Both too few and too many rules decrease the results. In our simulations all input fuzzy variables include five terms (\( n = 2 \)) implying 13 singletons in the output variable. Correspondingly, the two-input tracker has three parameters \( k_i, k_2, k_f \), the time-shift term \( s \) and 9 output singletons (\( n = 2 \)). As shown in (14), the alpha tracker can be considered a version of a single input fuzzy tracker with the single adjustable parameter \( \alpha \). No time-shift is used in the case of the alpha tracker.

In all cases the values of the parameters depend on the number of paths \( L \), on the velocity \( v \) of the...
mobile, on the signal-to-noise ratio SNR and on the number of the competing users \( K \). The signal structure of the last ones is not utilized in this paper but they are only considered as a type of noise.

To compare the different type trackers the parameters are numerically optimized to minimize the mean squared error (MSE)

\[
MSE = \frac{1}{NL} \sum_{n=0}^{L} \sum_{i=1}^{N} e_r(i)^2
\]

in which \( L \) is the number of paths, \( N \) is the number of data symbols and \( e_r(\cdot) \) is the error term:

\[
|e_r(i)| = \min \{ |c_r(i) - \hat{c}_r(i)|, |c_r(i) + \hat{c}_r(i)| \}
\]

6.1 Three-and two input model

First the three-input fuzzy tracker with triangular membership functions is optimized in the case of \( SNR = 0, 5, 10, 15, 20 \) dB, \( L = 1, 2, 3, 4 \), \( v = 50, 80, 100 \) km/h and \( s = 1, 2, 3, \ldots, 24 \). All the \( p \)ath powers are equal (0 dB). Based on these measurements, the dependence of the parameters \( k_1, k_2, k_3, k_f \) on \( SNR, L, v \) and \( K \) is illustrated by the limited four-variable linear model:

\[
k_1 = 0.3800 - 0.0063SNR - 0.2507L
+ 0.0290v - 0.1440K (0.1 \leq k_1 \leq 1)
\]

(17)

The value of \( k_2 \) is kept constant \( k_2 = 1 \) because the ratio \( k_1/k_2 \) is more essential than the absolute values of \( k_1 \) and \( k_2 \). For \( k_3 \) and \( k_f \) we have

\[
k_3 = 1.1800 - 0.0163SNR - 0.2507L
+ 0.0290v - 0.1440K (0.1 \leq k_3 \leq 2)
\]

(18)

\[
k_f = 0.0280 - 0.0004SNR - 0.0045L
+ 0.0007v - 0.0010K (0.1 \leq k_f \leq 1)
\]

(19)

Finally

\[
s = \left[ 22.35 - 0.1383SNR + 0.0553L \right] + 0.004v + 1.0800K
(0 \leq s \leq 24)
\]

(20)

To test the influence of the feedback term in the three-dimensional model we compared its results to

the two-input filter which was obtained by omitting \( k_3 \).

6.2 Single-input model (alpha tracker)

The alpha tracker is optimized independently by using the same test data as in the case of the three-input filter. For alpha we estimated the formula

\[
\alpha = 0.8767 - 0.0060SNR + 0.0553L
- 0.0040v + 0.0140K (0.1 \leq \alpha \leq 0.9)
\]

(21)

6.3 Results

All three-type of trackers were tested in the area of \( 0 \leq SNR \leq 20 \) dB, \( 0 \leq L \leq 4 \), each path equal power 0 dB, \( 50 \leq v \leq 100 \) km/h and \( 5 \leq K \leq 15 \). In each tested \((SNR, L, v, K)\)-cell 10000 bits (20 frames) were used.

All the three trackers gave similar results in order of magnitude. However, we can see that the two-input model is the best one in relative noisy circumstances, say \( 0 \leq SNR \leq 10 \) dB. For \( 10 < SNR \leq 20 \) dB the single input model i.e. the adaptive alpha tracker is the winner. In all cases the three-input model gave worse results than the another ones.

<table>
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Table 1 Two-input fuzzy tracker, 15 users, each path 0 dB for all users, \( v = 100 \) km/h

<table>
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<td>0.0446</td>
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Table 2 Alpha tracker (single input fuzzy tracker) , 15 users, each path 0 dB for all users, \( v = 100 \) km/h

\[ [x] \] is the smallest integer less or equal than \( x \).
Among the considered three type trackers the two- and single input models proved to be the most accurate systems in the considered circumstances (Rayleigh fading CDMA channel with AWGN). In the three input model the feedback term did not introduced any benefits to the results. In [4] we used filters with fixed parameters together with the moving average of ten coefficients. By using the adjustable parameters the moving average is not needed and still the results improved.

The fact that the single input system is a special case of the two-input one makes possible easily to construct a 'hybrid' filter which changes its role according to the noise circumstances: in noisy situations the change in error input will be switch on and in clear situations off.

### References:


### Table 3 Two-input fuzzy tracker, 5 users, each path 0 dB for all users, v= 50 km/h

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### Table 4 Alpha tracker (single input fuzzy tracker) , 5 users, each path 0 dB for all users, v= 50 km/h

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<td>0.0046</td>
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</table>

### 7 Conclusion

Among the considered three type trackers the two- and single input models proved to be the most accurate systems in the considered circumstances (Rayleigh fading CDMA channel with AWGN). In the three input model the feedback term did not introduced any benefits to the results. In [4] we used filters with fixed parameters together with the moving average of ten coefficients. By using the adjustable parameters the moving average is not needed and still the results improved.