Abstract: - Theoretical investigation of the thermal heat transfer associated with the biological tissues is important for understanding the deviation of the experimental results from the theoretical analysis. In the present work, an analytical modeling of finding the temperature distribution inside the biological tissues due to the flow of blood is done. The tissues are approximated as a single body using the continuum approach with one side extending to infinity. Due to flow of blood on the remaining two sides there is a convective heat transfer associated with the walls of the tissues. Volumetric heating due to the metabolic processes is considered and integral energy equation is used as the governing equation. Appropriate boundary conditions are used to find the steady state temperature profile inside the biological tissues. The second order temperature profile, thus obtained, is substituted in the integral equation and the final transient state temperature distribution is evaluated on solving the integral equation. The final temperature distribution shows the transient response of the tissues due to blood flow and has the exponential dependence on the time involved.

Key-Words: - Integral approach, Energy conservation, Steady-state solution, Transient analysis, Exponential dependence, Temperature profile

1 Introduction
Thermal analysis of the biological tissues is critical for better understanding of heat transfer phenomena inside the tissues. Different experimental conditions have been established [1]-[4] which studies the heat transfer phenomena inside the biological tissues. The objective of the present work is to present an analytical modeling of the temperature distribution inside the biological tissues due to the flow of blood in the surrounding domain. Continuum approach is used in one-dimensional to model the tissues as a single body, extending the other side to infinite region. Due to the flow of blood around the tissues heat transfer associated with convection plays a critical role. The energy conservation equation in the form of integral equation is used as the governing equation. Since the temperature distribution is dependent on the position as well as time hence the temperature profile is assumed to a product of two independent profiles, one dependent on the distance (x) while the other on the time (t). The x-dependence solution is obtained using the steady-state heat diffusion equation and applying necessary boundary conditions. This profile is substituted in the governing equation to find the t-dependence of the temperature after applying appropriate boundary conditions. Finally the x and t temperature distribution are merged together to result in the final expression for the temperature distribution inside the tissues.

2 Problem Formulation
Consider the tissues in contact with the blood be modeled as a single body as shown in Fig. 1. \( W_i \) is the volumetric heating per unit length inside the tissues generated due to the metabolic processes. \( T_f \) represents the temperature of the surrounding fluid and \( h \) is the convective heat transfer coefficient.
Due to the symmetry only half of the tissues can be considered for analytical modeling as shown in Fig. 2.

3 Analytical Approach
The energy balance equation for the above case is given by:

\[ \dot{S} = \dot{I} - \dot{O} \]

where \( \dot{S} \) is the energy storage rate, \( \dot{I} \) is the rate of energy input and \( \dot{O} \) is the rate of energy output given by the following equations respectively:

\[ \dot{S} = \rho c \frac{d}{dt} \int_{0}^{L} (T - T_{f}) \, dx, \]

\[ \dot{I} = W_{i} L \]

\[ \dot{O} = -k \cdot \frac{1}{x_{k=L}} \left( T - T_{f} \right) \]  
(Fourier's Law of diffusion)

where \( \rho \), \( c \) and \( k \) represent the density, specific heat and the thermal conductivity of the tissue body. Substituting the above terms the governing equation for the problem in consideration is given by:

\[ \rho c \frac{d}{dt} \int_{0}^{L} (T - T_{f}) \, dx = W_{i} L + k \cdot \frac{1}{x_{k=L}} \left( T - T_{f} \right) \]

(1)

The temperature profile is assumed to be of the following form [5]:

\[ T - T_{f} = \theta(t) X(x) \]

(2)

which takes into account the dependence on temperature both on the distance and the time. Here the first term (I) signifies the temperature dependence on the time and the second term (II) signifies the dependence on the distance. Since the second term has only the linear distance dependence it can be solved using one-dimensional heat diffusion equation given by:

\[ \frac{\partial^{2} T}{\partial x^{2}} + \frac{W_{i}}{k} = 0 \]

Applying the boundary conditions:

1) \( x = 0, \frac{T}{x} = 0 \), and

2) \( x = L, W_{i} L = h \left( T_{f} - T_{i} \right) \), or,

\[ T_{x=L} = T_{f} + \frac{W_{i} L}{h} \]

Substituting the above boundary conditions the \( x \)-dependence of temperature is of the form:

\[ X(x) = \frac{W_{i} L}{h} + \frac{W_{i} L^{2}}{2k} \left(1 - \frac{x^{2}}{L^{2}}\right) \]

Substituting the value of \( X(x) \) in the Eq. (1) and rearranging the terms we get:

\[ \frac{d}{dt} \int_{0}^{L} \left[ \frac{W_{i} L}{h} + \frac{W_{i} L^{2}}{2k} \left(1 - \frac{x^{2}}{L^{2}}\right) \right] \theta(t) \, dx = \frac{W_{i} L}{\rho c} \]

or,

\[ \frac{d}{dt} \int_{0}^{L} \left[ -\frac{W_{i} L^{2}}{2k} \left(1 - \frac{2L}{L^{2}}\right) \right] \theta(t) \, dx = \frac{W_{i} L}{\rho c} \]
\[
\frac{W_i L^2}{2k} \frac{d}{dt} \int_0^L \left[ \frac{2k}{hL} \theta(t) + \left( 1 - \frac{x^2}{L^2} \right) \theta(t) \right] dx = \frac{W_i L}{ \rho c} \left[ 1 - \theta(t) \right]
\]

alternatively,

\[
\frac{d}{dt} \left[ \frac{2k}{h} \theta(t) + \frac{2L}{3} \theta(t) \right] = \frac{2\alpha}{L} \left[ 1 - \theta(t) \right] \quad (3)
\]

where \( \alpha \) is the thermal diffusivity of the tissue given by:

\[
\alpha = \frac{k}{\rho c}
\]

Rewriting the Eq. (3) in the form of linear differential equation operating on \( \theta(t) \) as:

\[
\frac{d\theta}{dt} + \frac{\alpha}{\left[ \frac{k}{h} + \frac{L^2}{3} \right] L} \theta - \frac{\alpha}{\left[ \frac{k}{h} + \frac{L^2}{3} \right] L} = 0 \quad (4)
\]

Using the initial condition as:

\[\text{at } t = 0, \; T_f = T_i \quad (5)\]

where \( T_i \) is the initial temperature equal to the fluid temperature.

Solving the differential equation given by Eq. (4) using the initial condition given by Eq. (5) for \( \theta \), we get:

\[
\theta(t) = 1 - \exp \left( -\frac{\alpha t}{\left[ \frac{k}{h} + \frac{L^2}{3} \right] L} \right)
\]

Using the temperature profile given by Eq. (2), we get:

\[
T - T_f = \left[ \frac{W L}{h} + \frac{W_i L^2}{2k} \left( 1 - \frac{x^2}{L^2} \right) \right] \left[ 1 - \exp \left( -\frac{\alpha t}{\left[ \frac{k}{h} + \frac{L^2}{3} \right] L} \right) \right]
\]

The above equation represents the temperature distribution inside the tissues due to the flow of blood as the ambient conditions. The equation shows that the temperature has an exponential dependence on the time.

### 4 Conclusion

Analytical modeling of the thermal heat transfer associated with the biological tissues due to the hydrodynamics of blood flow is done. The steady-state temperature profile is derived to find the temperature dependence on the distance. Using the steady state temperature profile and the convection heat transfer phenomenon as the boundary condition in the governing integral equation the equation for temperature dependence on time is determined. Final temperature distribution is obtained by combining the distance and time dependence.

### References:


