Abstract: - This paper presents a new model for the performance analysis of IEEE 802.11 saturation throughput with freezing of the backoff counter. The model corrects the exiting model discussed by Foh and Tantra. Simulation results show the accuracy of the new model.

Key-Words: - Wireless local area network, Performance analysis, Saturation throughput analysis

1 Introduction

The bi-dimensional Markov chain modeling introduced by Bianchi [1] for the analysis of the IEEE 802.11 saturation throughput has become a common method to study the performance of the IEEE 802.11 Medium Access Control (MAC) protocol [2] and its enhancements. The model was later refined to capture further details of the IEEE 802.11 protocol operations. Among the refinements, one is due to Ziouva and Antonakopoulos [3] aiming to capture the freezing of backoff counters when the broadcast channel is sensed busy by a station. Precisely, when a channel turns idle from busy due to, for example, a Distributed Inter-Frame Space (DIFS), Bianchi’s model assumes that each station immediately reactivates and decrements its counter, whereas the IEEE 802.11 standard specifies that a backoff counter is decremented only after the channel continues to remain idle for a predefined slot time. The refinement reported in [3] was, however, introduced without realizing that the two key probabilities governing the performance, namely the channel access probability \( \tau \) and the station collision probability \( p \), depend on the channel status. Foh and Tantra [4] presented a model correcting that of [3] by evaluating the channel access probabilities and the station collision probabilities conditioned upon whether the previous period is idle, busy due to a successful transmission or busy due to \( k \) consecutive collisions. We then show the accuracy of our results via computer simulation, and demonstrate the errors if such details are ignored.

2 Saturation Throughput Analysis

The mechanism of the IEEE 802.11 Distributed Coordination Function (DCF) with basic access method differs from the model presented in [1] in the decrement of the backoff counter. The IEEE 802.11 standard [2] specifies that a station freezes its backoff counter when it detects a transmission on the channel. This backoff freezing procedure directly affects the channel access probability and the station collision probability, and these probabilities also depend on whether the previous period is idle, busy due to a successful transmission or busy due to \( k \) consecutive collisions. To understand this, we first consider the channel access event after a busy period due to a collision. After an idle slot time, all stations whose backoff counters are decremented to zero will access the channel. As opposed to the case of an idle slot time, after a collision, since stations that did not participate in this collision had frozen their backoff counters, they will not access the channel just after the busy period; only those suffered a collision may access the channel if their newly chosen backoff counter is zero. After \( k \)
consecutive collisions, only those suffered all of \( k \) consecutive collisions may access the channel if their newly chosen backoff counter is zero again. Hence, the more the number of consecutive collisions is the less the number of stations that may access the channel after the consecutive busy periods is. In case of successful transmission, only one station, which performed the successful transmission, may access the channel just after the successful transmission period. Hence, the probability that a station detects a transmission on the channel after a busy period due to a successful transmission is different from the probability that a station detects a transmission on the channel after a busy period due to a collision; therefore, it is obvious that the channel access probability and the station collision probability actually depend on whether the previous period is idle, busy due to a successful transmission or busy due to \( k \) consecutive collisions. This is not modeled in [4] where the derived channel access probability and the derived station collision probability are conditioned upon only whether the previous period is idle or busy.

We consider a fixed number \( n \) of contending stations. In saturated conditions, each station has immediately a packet available for transmission. Let \( s(t) \) be the stochastic process representing the backoff stage of a given station at the beginning of slot time \( t \) and let \( b(t) \) be the stochastic process representing the backoff counter for the tagged station at the beginning of slot time \( t \). Let \( c(t) \) denote the stochastic process representing the following:

\[
e(t) = \begin{cases} 
-2 & \text{if the tagged station performed a successful transmission during the previous period}, \\
-1 & \text{if the previous period is busy due to a successful transmission of the other stations}, \\
0 & \text{if the previous period is idle}, \\
\text{k} & \text{if the previous period is busy due to } k, k > 0, \text{ consecutive collisions}.
\end{cases}
\]

The tri-dimensional process \( \{s(t), b(t), c(t)\} \) is a discrete-time Markov chain presented in Figure 1. The maximum backoff stage is denoted by \( m \) and the backoff window of a station at the \( i \)th backoff stage is \( W_i \).

Key probabilities \( p_{k0} \) and \( p_{k1} \) governing the backoff process are first defined. The probability \( p_{k0} \) (resp., \( p_{k1} \)) is the probability that, from the tagged station’s point of view, at least two (resp., only one) of the other stations transmit during a slot time after \( k - 1 \) consecutive collisions. We further define

\[
p_k = p_{k0} + p_{k1}. \tag{1}
\]

Let \( b_{i,j,k} \) be the stationary distribution of the described Markov chain. Owing to the chain regularity, the following relations hold:

\[
b_{0,j,-2} = b_{0,0,-2}, \quad 1 \leq j < W_0,
\]
\[
b_{0,0,0} = \frac{W_0 - 1}{W_0} b_{0,0,-2},
\]
\[
b_{i,0,0} = \frac{W_i - 1}{W_i} \sum_{k=0}^{i-1} p_{k+1} b_{i-1,0,k}, \quad 1 \leq i < m,
\]
\[
b_{i,0,k} = \frac{p_k}{W_i} b_{i-1,0,k-1}, \quad 1 \leq k \leq i < m,
\]
\[
b_{m,0,0} = \frac{W_m - 1}{W_m} \sum_{k=0}^{m-1} b_{m-1,0,k} \prod_{r=k+1}^{l+1} \frac{p_r}{W_m},
\]
\[
1 - (W_m - 1) \sum_{r=1}^{l+1} \frac{p_r}{W_m}, \quad k \geq 1,
\]
\[
b_{m,0,k} = \sum_{l=0}^{\min(k,m)-1} b_{m-1,0,l} \prod_{r=l+1}^{k} \frac{p_r}{W_m} + b_{m,0,0} \prod_{r=1}^{k} \frac{p_r}{W_m}, \quad k \geq 1,
\]
\[
b_{i,j,0} = \frac{W_i - j - 1}{W_i - 1} b_{i,0,0}, \quad 0 \leq i < m, \quad 1 \leq j < W_i,
\]
\[
b_{i,j,k} = \frac{p_k}{W_i} b_{i-1,0,k-1} + p_{k0} b_{i,j,k-1}, \quad 0 \leq i < m, \quad 1 \leq j < W_i, \quad 1 \leq k \leq i,
\]
\[
b_{i,j,k} = p_{k0} b_{i,j,k-1}, \quad 0 \leq i < m, \quad 1 \leq j < W_i, \quad i < k,
\]
\[
b_{m,j,k} = \frac{p_k}{W_m} \left( b_{m-1,0,k-1} + b_{m,0,k-1} \right) + p_{k0} b_{m,j,k-1}, \quad 1 \leq j < W_m, \quad 1 \leq k \leq m,
\]
\[
b_{m,j,k} = \frac{p_k}{W_m} b_{m,0,k-1} + p_{k0} b_{m,j,k-1}, \quad 1 \leq j < W_m, \quad k > m,
\]
\[
b_{i,j,-1} = \frac{W_0 - 1}{W_0} \sum_{k=0}^{\infty} p_{k+1} b_{i,j,k},
\]
\[
0 \leq i < m, \quad 1 \leq j < W_i.
\]

Define \( \tau_k \) to be the probability that a station accesses the broadcast channel again after \( k - 1 \) consecutive collisions. The probability \( \tau_k \) can be expressed as a function of the stationary probabilities. They are given by

\[
\tau_k = \begin{cases} 
\frac{\sum_{i=0}^{m-k-1} b_{i,k-1}}{\sum_{i=0}^{m-k-1} \sum_{j=0}^{m-k-i} b_{i,j,k-1}}, & \text{if } 0 \leq k < m, \\
\frac{\sum_{i=0}^{m-k-1} b_{i,k-1}}{\sum_{j=0}^{m-k-i} b_{m,j,k-1}}, & \text{if } k \geq m.
\end{cases} \tag{2}
\]

Having obtained \( \tau_k \), we have

\[
p_{k0} = 1 - (1 - \tau_k)^{n-1} - (n - 1) \tau_k (1 - \tau_k)^{n-2}, \tag{3}
\]
\[
p_{k1} = (n - 1) \tau_k (1 - \tau_k)^{n-2}. \tag{4}
\]
Figure 1. State transition diagram of IEEE 802.11 DCF
The system throughput $S$, the fraction of time used for successful payload transmission, can be expressed as

$$S = \left[ b_{0,0,-2} + \sum_{i=0}^{m-1} \sum_{k=0}^{i} (1 - p_{k+1})b_{i,0,k} \right. $$

$$+ \sum_{k=0}^{\infty} (1 - p_{k+1})b_{m,0,k}$$

$$+ \sum_{i=0}^{m} \sum_{j=1}^{W_i-1} \left\{ \frac{1}{W_0}b_{m,j-1} + \sum_{k=0}^{\infty} p_{k+1,1}b_{i,j,k} \right\}$$

$$\times E[P] \sum_{i,j,k} b_{i,j,k}h_{i,j,k}$$

where $E[P]$ is the average payload length and $h_{i,j,k}$ is the
mean sojourn time at state $(i, j, k)$.

The mean sojourn times $h_{i,j,k}$ can be expressed as

$$h_{0,0,-2} = T_s,$$

$$h_{0,j,-2} = \sigma, \quad 1 \leq j < W_0,$$

$$h_{i,0,k} = p_{k+1}T_s + (1 - p_{k+1})T_s, \quad 0 \leq k \leq i < m,$$

$$h_{m,0,k} = p_{k+1}T_s + (1 - p_{k+1})T_s, \quad k \geq 0$$

$$h_{i,j,k} = p_{k+1,0}T_c + p_{k+1,1}T_s + (1 - p_{k+1})\sigma,$$

$$0 \leq i \leq m, \quad 1 \leq j < W_i, \quad k \geq 0,$$

$$h_{i,j,-1} = \frac{1}{W_0}T_s + \frac{W_0 - 1}{W_0}\sigma,$$

$$0 \leq i \leq m, \quad 1 \leq j < W_i,$$

where $\sigma$ is the duration of an empty slot time, $T_s$ is the average time that the channel is sensed busy because of a successful transmission, $T_c$ is the average time that the channel is sensed busy due to a collision. These quantities for the basic and the RTS-CTS access methods are given in [1].

3 Numerical Results

In Figure 2, numerical results for the saturation throughput obtained from our model are plotted and compared with that of [4] when $m = 2, W_0 = 8, W_1 = 16$ and $W_2 = 32$. We use the same protocol parameters as [1] for this comparison. The numerical results are obtained using the fixed point iteration technique [5]. Figure 3 shows the difference between the simulation results and the analysis based on the proposed model for $m = 7, CW_{\text{min}} = 32, CW_{\text{max}} = 1024$ and $W_i = \min(2^{\text{th}}, CW_{\text{min}}, CW_{\text{max}})$. As can be seen in Figure 2 and Figure 3, our model gives accurate results for the performance of IEEE 802.11 with freezing of backoff counters.

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References


