Abstract: The Brushless Direct Current (BLDC) motor is one of the motors types rapidly gaining attractiveness. Actually they are becoming widely used in various consumer and industrial systems. As the name implies, BLDC motors do not use brushes for commutation, they are electronically commutated. This paper presents a Matlab simulation model that includes the control speed and position of a trapezoidal back-emf BLDC motor. The model consists of position and speed linear controllers, reference current generation, current controller and a three phase voltage inverter. Most of the applications now use a Pulse Width Modulation (PWM) current controller. The solution here proposed it new because it uses a hysteretic vectorial control in $\alpha\beta$ to control the current.

Key-Words: BLDC motor, Position control, Speed control, Current control.

1 Introduction
A brushless DC motor is a DC motor with one particularly modification, the field is on the rotor and the armature is on the stator. The brushless DC motor is actually a permanent magnet ac motor whose torque-current characteristics mimic the DC motor. Instead of commutating the armature current using brushes, electronic commutation is used [1]. This eliminates the problems associated with the brush and the commutator elements, for example, sparking and wearing out of the commutator-brush elements increasing its efficiency due to reduced losses, low maintenance and low rotor inertia, thus, making a BLDC more robust as compared to a DC motor [2].

In effect, a BLDC is a modified Permanent Magnet Synchronous Motor (PMSM) motor with the modification being that the back-emf is trapezoidal instead of being sinusoidal as in the case of PMSM[1]. These two designs eliminate the rotor copper losses, giving very high peak efficiency compared with a traditional induction motor (around 95 % and more in Nd-Fe-B machines in the 20 to 100 kW range). Besides, the power-to-weight ratio of PMSM and BLDC motor is higher than the equivalent squirrel cage induction machines. The aforementioned characteristics and a high reliability control make this type of machine a powerful traction system [3]. However, sensing the phase currents and the rotor position are the two main drawbacks of this machine.

The general type of position sensing for the BLDC motor uses to Hall-effect position sensors to detect the flux distributions of rotor magnets. Fig. 1 illustrates the structure of a typical three-phase brushless DC motor.

![Fig. 1. Disassembled view of a brushless DC motor](image)

This paper presents a Matlab simulation model that uses linear controllers coupled to hysteretic vectorial $\alpha\beta$ current controllers, to achieve speed and position control of the BLDC motor. The basic building blocks of the system are described. The paper is organized as follows. The description of the whole system is in Section 2. The mathematical model of the BLDC motor is developed in Section 3. The operation of the hysteretic vectorial current method control is presented in Section 4. Results and conclusions are in Sections 5 and 6, respectively.
2 System descriptions
Fig. 1 shows the block diagram of the drive system under consideration.

![Fig. 2. System block diagram](image)

The position of the motor, $\theta_r$, is compared with its reference value, $\theta_{ref}$, and the position error is processed by a proportional-integral (PI) controller. This controller generates the speed reference that will be compared with the actual speed. The speed error is also processed by a PI controller. The output of this second controller is considered as the reference torque, $T^*$. A limit block is placed on the speed controller output. The torque will originate the value of the reference current. The reference current generator block, generates the three phase currents, $i_a^*$, $i_b^*$, $i_c^*$ using the reference current decided by the controller and the rotor position. These currents have the shape of quasi-square wave in phase with the respective back-emf to develop constant unidirectional torque. The next block converts the three-phase reference currents in $\alpha\beta$ through the Concordia matrix. The current controller regulates the winding currents in $\alpha\beta$ within the small band around reference currents. The motor currents are compared with the reference currents and the switching commands are generated by the inverter characteristics.

3 Basic Principle of BLDC Motor
The BLDC has three stator windings and permanent magnets on the star connect rotor with no accessible neutral point. The block diagram of the BLDC motor drive is show in Fig. 3.

The analysis is based on the following assumptions [5]:

- The motor is not saturated
- Stator resistances of all windings are equal and self mutual inductances are constant.
- Power semiconductors devices in the inverter are ideal.

![Fig. 3. Configuration of BLDC motor drive system](image)

Under the above considerations, the circuit equations of the three windings in phase variables are given by (1).

$$
\begin{bmatrix}
v_a \\
v_b \\
v_c \\
\end{bmatrix}
= 
\begin{bmatrix}
\frac{L_a}{R_a} & 0 & 0 \\
0 & \frac{L_b}{R_b} & 0 \\
0 & 0 & \frac{L_c}{R_c} \\
\end{bmatrix}
\begin{bmatrix}
i_a \\
i_b \\
i_c \\
\end{bmatrix}
+ 
\begin{bmatrix}
\frac{L_{ab}}{R_a} & \frac{L_{ba}}{R_b} & \frac{L_{ca}}{R_c} \\
\frac{L_{ac}}{R_a} & \frac{L_{cb}}{R_b} & \frac{L_{bc}}{R_c} \\
\frac{L_{ac}}{R_a} & \frac{L_{bc}}{R_b} & \frac{L_{cb}}{R_c} \\
\end{bmatrix}
\frac{d}{dt}
\begin{bmatrix}
i_a \\
i_b \\
i_c \\
\end{bmatrix}
+ 
\begin{bmatrix}
K_t \\
K_t \\
K_t \\
\end{bmatrix}
\begin{bmatrix}
e_a \\
e_b \\
e_c \\
\end{bmatrix}
(1)
$$

In (1), $R_a$, $R_b$ and $R_c$ are the resistances of each winding, $e_a$, $e_b$ and $e_c$ are the trapezoidal shaped back-emfs and $i_a$, $i_b$ and $i_c$ are the rectangular shaped currents as show in Fig. 4.

![Fig. 4. Back-emf and Phase Current Waveforms](image)

As show in Fig. 4, the back-emfs and currents are function of rotor position $(\theta_r)$, and they have the amplitude of $E = K_e \omega_r$, ($K_e$ is the back-emf constant) and the reference current amplitude is $I^* = T^*/K_e$. Tables 1 and 2 show how the waves are generated.
Table 1. Trabezoidal back-emf function

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>( e_a )</th>
<th>( e_b )</th>
<th>( e_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-30°</td>
<td>( (6E/\pi)\theta )</td>
<td>-E</td>
<td>E</td>
</tr>
<tr>
<td>30°-60°</td>
<td>E</td>
<td>-E</td>
<td>( -6E/\pi \theta + 2E )</td>
</tr>
<tr>
<td>60°-90°</td>
<td>E</td>
<td>-E</td>
<td>( -6E/\pi \theta + 2E )</td>
</tr>
<tr>
<td>90°-120°</td>
<td>E</td>
<td>( (6E/\pi)\theta - 4E )</td>
<td>-E</td>
</tr>
<tr>
<td>120°-150°</td>
<td>E</td>
<td>( (6E/\pi)\theta - 4E )</td>
<td>-E</td>
</tr>
<tr>
<td>150°-180°</td>
<td>( -6E/\pi \theta + 6E )</td>
<td>E</td>
<td>-E</td>
</tr>
<tr>
<td>180°-210°</td>
<td>( -6E/\pi \theta + 6E )</td>
<td>E</td>
<td>-E</td>
</tr>
<tr>
<td>210°-240°</td>
<td>-E</td>
<td>E</td>
<td>( (6E/\pi)\theta - 8E )</td>
</tr>
<tr>
<td>240°-270°</td>
<td>-E</td>
<td>E</td>
<td>( (6E/\pi)\theta - 8E )</td>
</tr>
<tr>
<td>270°-300°</td>
<td>-E</td>
<td>( -6E/\pi \theta + 10E )</td>
<td>E</td>
</tr>
<tr>
<td>300°-330°</td>
<td>-E</td>
<td>( -6E/\pi \theta + 10E )</td>
<td>E</td>
</tr>
<tr>
<td>330°-360°</td>
<td>( (6E/\pi)\theta + 12E )</td>
<td>-E</td>
<td>E</td>
</tr>
</tbody>
</table>

Table 2. Reference current generation

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>( i_a^* )</th>
<th>( i_b^* )</th>
<th>( i_c^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-60°</td>
<td>( i^* )</td>
<td>-( i^* )</td>
<td>0</td>
</tr>
<tr>
<td>60°-120°</td>
<td>( i^* )</td>
<td>0</td>
<td>-( i^* )</td>
</tr>
<tr>
<td>120°-180°</td>
<td>0</td>
<td>( i^* )</td>
<td>-( i^* )</td>
</tr>
<tr>
<td>180°-240°</td>
<td>-( i^* )</td>
<td>( i^* )</td>
<td>0</td>
</tr>
<tr>
<td>240°-300°</td>
<td>-( i^* )</td>
<td>0</td>
<td>( i^* )</td>
</tr>
<tr>
<td>300°-360°</td>
<td>0</td>
<td>-( i^* )</td>
<td>( i^* )</td>
</tr>
</tbody>
</table>

Assuming that there is no change in the rotor reluctance with angle [6], can be considered that \( L_a = L_b = L_c = L \), \( R_a = R_b = R_c = R \) and \( L_{ab} = L_{ba} = L_{bc} = L_{cb} = L_{ca} = L_{ac} = M \).

Considering also that, \( i_a + i_b + i_c = 0 \), the phase windings currents can be expressed in (2)

\[
\begin{bmatrix}
\frac{d}{dt} i_a \\
\frac{d}{dt} i_b \\
\frac{d}{dt} i_c
\end{bmatrix} =
\begin{bmatrix}
1/(L-M) & 0 & 0 \\
0 & 1/(L-M) & 0 \\
0 & 0 & 1/(L-M)
\end{bmatrix}
\begin{bmatrix}
v_a \\
v_b \\
v_c
\end{bmatrix} + \begin{bmatrix}
R & 0 & 0 \\
0 & R & 0 \\
0 & 0 & R
\end{bmatrix} \begin{bmatrix}
i_a \\
i_b \\
i_c
\end{bmatrix} + \begin{bmatrix}
e_a \\
e_b \\
e_c
\end{bmatrix}
\]

In this kind of machine where only two of the three phases are excited through the conduction operating modes, the three-phase voltages are considered in terms of line-to-line voltages. From Fig. 5, the (3) and (4) voltage and current equations can be obtained.

Table 3. Motor specifications

<table>
<thead>
<tr>
<th>( K_F )</th>
<th>0.049V/(rad/sec)</th>
<th>( L )</th>
<th>2.72 ×10^{-3}H</th>
</tr>
</thead>
<tbody>
<tr>
<td>( J )</td>
<td>0.0002kg.m²</td>
<td>( B )</td>
<td>0.002 Nms</td>
</tr>
<tr>
<td>( R )</td>
<td>0.75Ω</td>
<td>( M )</td>
<td>-1.5×10^{-3}H</td>
</tr>
</tbody>
</table>

3 Current Controller Design

The Fig. 6 shows the block diagram of the implemented current controller.
3.1 The Three-phase Inverter

From Fig. 3, the three-phase voltages $v_{an}$, $v_{bn}$ and $v_{cn}$ is depend on the semiconductor state of each leg. The condition of the semiconductors is represented by $\gamma_k$, where $k = (a, b, c)$ and can be written as presented in (7).

$$\gamma_k = \begin{cases} 1 & \text{if } S_{1k} \text{ on and } S_{2k} \text{ off} \\ 0 & \text{if } S_{1k} \text{ off and } S_{2k} \text{ on} \end{cases}$$

(7)

Hence, the voltages $v_a$, $v_b$ and $v_c$ are expressed by (8).

$$v_k = \gamma_k \cdot U$$

(8)

The line-to-line voltages are given by (9).

$$v_{ab} = v_a - v_b = (\gamma_a - \gamma_b) \cdot U = v_{an} - v_{bn}$$

$$v_{bc} = v_b - v_c = (\gamma_b - \gamma_c) \cdot U = v_{bn} - v_{cn}$$

$$v_{ca} = v_c - v_a = (\gamma_c - \gamma_a) \cdot U = v_{cn} - v_{an}$$

(9)

Considering (10) and operating (10) and (9), results in (11)

$$v_{an} + v_{bn} + v_{cn} = 0$$

(10)

$$v_{an} = \frac{2\gamma_a - \gamma_b - \gamma_c}{3} \cdot U$$

$$v_{bn} = \frac{2\gamma_b - \gamma_c - \gamma_a}{3} \cdot U$$

(11)

Converting to the $\alpha\beta$ referential (12) is obtained.

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & 0 \\ \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \gamma_a \\ \gamma_b \\ \gamma_c \end{bmatrix}$$

(12)

Depending on the $\gamma_k$ value, the voltages from the rectifier arms can assumed $2^3$ possible states, that are expressed in the $\alpha\beta$ referential as

$$v_i = v_{ai} + jv_{bi} = \sqrt{\frac{2}{3}} \cdot U \cdot \frac{\gamma(i-1)}{3} + (i = 1..6)$$

(13)

This expression in graphical terms is presented in Fig. (7).

Fig. 7. The 8 possible voltages vectors in $\alpha\beta$

3.2 Current Control Strategy

Since the neutral of the machine is not accessible, the sum of the three currents in the stator is always zero $i_a + i_b + i_c = 0$. Hence, the use of two sensors is enough, being the third current obtained by $i_c = -i_a - i_b$.

The current control can be improved if $\alpha\beta$ coordinates are used. Only two hysteretic comparators are needed, because the information corresponding to the homopolar component is not necessary. The comparators measured the error between the $\alpha\beta$ current reference and the $\alpha\beta$ current.

From Fig. 3, (13) is obtained.

$$L \frac{di_k}{dt} = v_{hn} - R \cdot i_k - e_k$$

Expression (13) is in $\alpha\beta$ is represented by (14).
To choose one of the 8 voltage vectors (Fig. 7), it is necessary that, each hysteretic comparator evaluates the error, \( e_{iaβ} \), between the \( αβ \) current reference, \( i_{aβref} \), and \( αβ \) current measured, \( i_{aβ} \), and quantizes the result in three discrete values [7], as presented in (15) and (16)

\[
e_{iaβ} = i_{aβref} - i_{aβ} \quad (15)
\]

\[
\begin{align*}
\delta_{a} > \Delta \Rightarrow & \delta_{a} = 1 \Rightarrow i_{aβ} > i_{aβref} \Rightarrow \frac{di_{aβ}}{dt} < 0 \Rightarrow v_{w} > 0 \\
\delta_{a} < -\Delta \Rightarrow & \delta_{a} = -1 \Rightarrow i_{aβ} < i_{aβref} \Rightarrow \frac{di_{aβ}}{dt} > 0 \Rightarrow v_{w} < 0 \\
\delta_{a} = 0 \Rightarrow & i_{aβ} = i_{aβref} \\
\end{align*}
\quad (16)
\]

In (16), \( \Delta \), represents the width of the hysteretic window, and \( \delta_{a} \), the comparators output.

The equation (16) applied to each \( e_{iaβ} \), leads to the 9 possible situations, summarized in table 4.

<table>
<thead>
<tr>
<th>( e_{ia} )</th>
<th>( e_{ib} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta_{a} = 1 )</td>
<td>( \delta_{b} = 1 )</td>
</tr>
<tr>
<td>( \delta_{a} = 0 )</td>
<td>( \delta_{b} = 0 )</td>
</tr>
<tr>
<td>( \delta_{a} = -1 )</td>
<td>( \delta_{b} = -1 )</td>
</tr>
</tbody>
</table>

Table 4. Vector strategy selection

Table 5 presents the voltage vector selection in function of the two and three levels comparators.

<table>
<thead>
<tr>
<th>( a )</th>
<th>( b )</th>
<th>( c )</th>
<th>( d )</th>
<th>( e )</th>
<th>( f )</th>
<th>( \text{Vector} )</th>
<th>( g )</th>
<th>( h )</th>
<th>( i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.5</td>
<td>-0.5</td>
<td>-0.5</td>
<td>-0.5</td>
<td>-1</td>
<td>-1</td>
<td>( v_{1} )</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0.5</td>
<td>-0.5</td>
<td>-0.5</td>
<td>-0.5</td>
<td>0</td>
<td>-1</td>
<td>( v_{3} )</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>-0.5</td>
<td>-0.5</td>
<td>1</td>
<td>-1</td>
<td>( v_{5} )</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>-0.5</td>
<td>-0.5</td>
<td>-0.5</td>
<td>-0.5</td>
<td>0</td>
<td>-1</td>
<td>( v_{5} )</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>-0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>-0.5</td>
<td>0</td>
<td>0</td>
<td>( \phi = 1 \Rightarrow v_{7} )</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>-0.5</td>
<td>1</td>
<td>0</td>
<td>( v_{1} )</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>-0.5</td>
<td>0</td>
<td>0</td>
<td>( \phi = 0 \Rightarrow v_{6} )</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>0.5</td>
<td>-0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0</td>
<td>1</td>
<td>( v_{2} )</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>-0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>-1</td>
<td>1</td>
<td>( v_{3} )</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>-0.5</td>
<td>-0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0</td>
<td>1</td>
<td>( v_{2} )</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

In Table 5, \( a=\delta_{La} \), \( b=\delta_{Na} \), \( c=\delta_{Lβ} \), \( d=\delta_{Nβ} \), \( e=\delta_{a} \), \( f=\delta_{β} \), \( g=\gamma_{a} \), \( h=\gamma_{b} \) and \( i=\gamma_{c} \).

More details about the current control strategy can be found in [7]-[8].

### 4 Simulation Results

To verify the proposed method, a simulation results are showed in Fig. 8. The simulation starts with a speed reference of 1500rpm, and change to -1500rpm at 0.25s. A load perturbation of 1Nm is added at 0.4s and 0.5Nm at 0.8s. The position reference of \( \pi \) rad occurs at 0.5s.

In Fig. 8(a), the PI controller forces the motor speed to follow the reference starting from 0.03s.
The response is smooth and there are no oscillations. It goes from 1500 to -1500rpm in 0.05s. The current waveforms, Fig 8 (b), are closed to the ones presented in Fig. 4.

5 Conclusions

The performance of a BLDC motor driven by a hysteretic vectorial controller in $\alpha\beta$, is considered in alternative to the PWM inverter [3,6,9], offering an efficient and faster alternative approach in the design of such drive system. The corresponding current, torque, position and speed performances using the proposed control method were simulated and validated. The proposed controller design, allows a straightforward application of these machines.

Future work includes experimental validation of this approach and comparison with another drive systems models and control techniques.

References:


